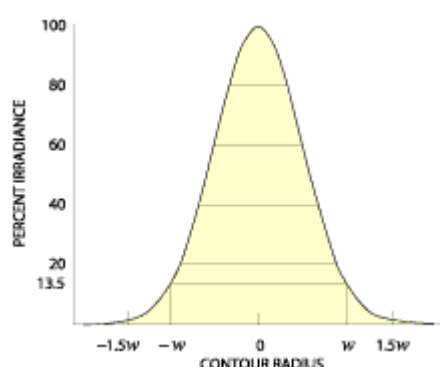


[Home](#)
[Optics Guide Contents](#)
[Fundamental Optics](#)
[Gaussian Beam Optics](#)
[Optical Specifications](#)
[Material Properties](#)
[Optical Coatings](#)
[Optics Glossary](#)
[Optics](#) → [Gaussian Beam Optics](#) → [Gaussian Beam Propagation](#) → [Beam Waist and Divergence](#)

Gaussian Beam Propagation

Beam Waist and Divergence

In order to gain an appreciation of the principles and limitations of Gaussian beam optics, it is necessary to understand the nature of the laser output beam. In TEM₀₀ mode, the beam emitted from a laser begins as a perfect plane wave with a Gaussian transverse irradiance profile as shown in the figure below. The Gaussian shape is truncated at some diameter either by the internal dimensions of the laser or by some limiting aperture in the optical train. To specify and discuss the propagation characteristics of a laser beam, we must define its diameter in some way. The commonly adopted definition is the diameter at which the beam irradiance (intensity) has fallen to $1/e^2$ (13.5%) of its peak, or axial, value.



Gaussian beam profile (theoretical TEM₀₀ mode)

Diffraction causes light waves to spread transversely as they propagate, and it is therefore impossible to have a perfectly collimated beam. The spreading of a laser beam is in precise accord with the predictions of pure diffraction theory; aberration is totally insignificant in the present context. Under quite ordinary circumstances, the beam spreading can be so small it can go unnoticed. The following formulas accurately describe beam spreading, making it easy to see the capabilities and limitations of laser beams.

Even if a Gaussian TEM₀₀ laser-beam wavefront were made perfectly flat at some plane, it would quickly acquire curvature and begin spreading in accordance with

$$R(z) = z \left[1 + \left(\frac{\pi w_0^2}{\lambda z} \right)^2 \right]$$

and

$$w(z) = w_0 \left[1 + \left(\frac{\lambda z}{\pi w_0^2} \right)^2 \right]^{1/2}$$

where z is the distance propagated from the plane where the wavefront is flat, λ is the wavelength of light, w_0 is the radius of the $1/e^2$ irradiance contour at the plane where the wavefront is flat, $w(z)$ is the radius of the $1/e^2$ contour after the wave has propagated a distance z , and $R(z)$ is the wavefront radius of curvature after propagating a distance z . $R(z)$ is infinite at $z = 0$, passes through a minimum at some finite z , and rises again toward infinity as z is further increased, asymptotically approaching the value of z itself. The plane $z = 0$ marks the location of a Gaussian waist, or a place where the wavefront is flat, and w_0 is called the beam waist radius.

The irradiance distribution of the Gaussian TEM₀₀ beam, namely,

$$I(r) = I_0 e^{-2r^2/w^2} = \frac{2P}{\pi w^2} e^{-2r^2/w^2},$$

where $w = w(z)$ and P is the total power in the beam, is the same at all cross sections of the beam. The invariance of the form of the distribution is a special consequence of the presumed Gaussian distribution at $z = 0$. If a uniform irradiance distribution had been presumed at $z = 0$, the pattern at $z = \infty$ would have been the familiar Airy disc pattern given by a Bessel function, while the pattern at intermediate z values would have been enormously complicated.

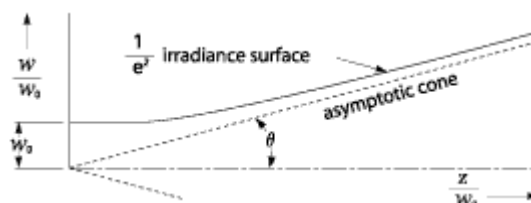
Simultaneously, as $R(z)$ asymptotically approaches z for large z , $w(z)$ asymptotically approaches the value

$$w(z) \cong \frac{\lambda z}{\pi w_0}$$

where z is presumed to be much larger than $\pi w_0^2 / \lambda$ so that the $1/e^2$ irradiance contours asymptotically approach a cone of angular radius

$$\theta \cong \frac{w(z)}{z} = \frac{\lambda}{\pi w_0}.$$

This value is the far-field angular radius (half-angle divergence) of the Gaussian TEM₀₀ beam. The vertex of the cone lies at the center of the waist, as shown in the figure below.



Growth in beam diameter as a function of distance from the beam waist

It is important to note that, for a given value of λ , variations of beam diameter and divergence with distance z are functions of a single parameter, w_0 , the beam waist radius.

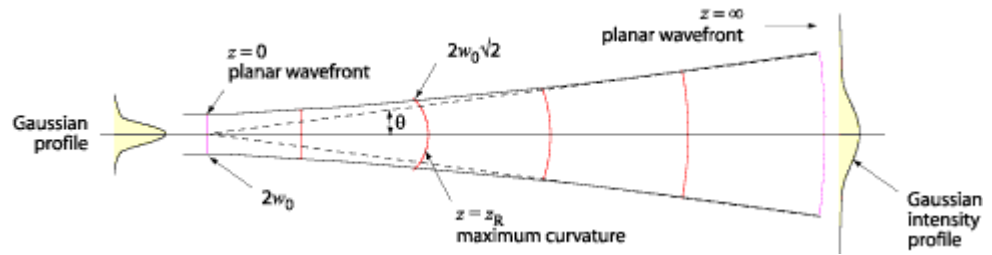
Near-Field vs. Far-Field Divergence

Unlike conventional light beams, Gaussian beams do not diverge linearly. Near the laser, the divergence angle is extremely small; far from the laser, the divergence angle approaches the asymptotic limit described above. The Rayleigh range (z_R), defined as the distance over which the beam radius spreads by a factor of the square-root of 2, is given by

$$z_R = \frac{\pi w_0^2}{\lambda}$$

At the beam waist ($z = 0$), the wavefront is planar ($R(0) = \infty$). Likewise, at $z = \infty$, the wavefront is planar ($R(\infty) = \infty$). As the beam propagates from the waist, the wavefront curvature, therefore, must increase to a maximum and then begin to decrease, as shown in the figure below. The Rayleigh range, considered to be the dividing line between near-field divergence and mid-range divergence, is the distance from the waist at which the wavefront curvature is a maximum. Far-field divergence (the number quoted in laser specifications) must be measured at a distance much greater than z_R (usually $>10 z_R$ will suffice). This is a very important distinction because calculations for spot size and other parameters in an optical train will be inaccurate if near- or mid-field divergence values are used. For a tightly focused beam, the distance from the waist (the focal point) to the far field can

be a few millimeters or less. For beams coming directly from the laser, the far-field distance can be measured in meters.



Changes in wavefront radius with propagation distance

[Back to Top](#)

[Previous](#) [Next](#)

Optics Guide © 2004 Melles Griot Inc.